Discuss the Ideas

1. For equations of the form $\sin x = c$, $\cos x = c$, and $\tan x = c$, where c is a constant, what restrictions are there on the value of c for the equations to have real solutions?

2. How do you recognize when an exact solution to a trigonometric equation exists?

3. A second-degree trigonometric equation in terms of $\sin x$, $\cos x$, or $\tan x$ is solved over the domain $0 \leq x < 2\pi$. How many roots could the equation have?

Exercises

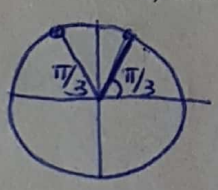
A

This is usually special angles.

Use algebra to solve each equation. Give exact values when possible; otherwise write the roots to the nearest degree or the nearest hundredth of a radian. Verify the solutions.

4. Solve each equation over the domain $0 \leq x < 2\pi$.

a) $\sin x = \frac{\sqrt{3}}{2}$ Where is $\sin x$ +'ve? IN QI & QII

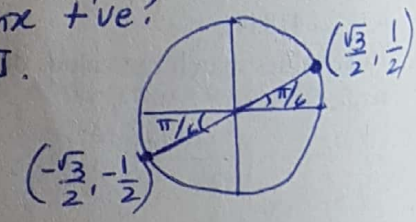


$x_R = \frac{\pi}{3}$ (reference angle)

In QI: $x = \frac{\pi}{3}$

In QII: $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

b) $\tan x = \frac{1}{\sqrt{3}}$ Where is $\tan x$ +'ve? IN QI & QIII.



$x_R = \frac{\pi}{6}$

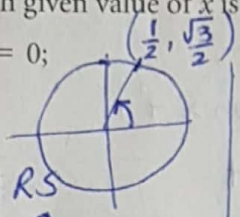
In QI: $x = \frac{\pi}{6}$

In QIII: $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

means to "check".

5. Verify that each given value of x is a root of the equation.

a) $\tan^2 x - 3 = 0$; $x = \frac{\pi}{3}$



L.S. | RS

$(\tan \frac{\pi}{3})^2 - 3 = 0$

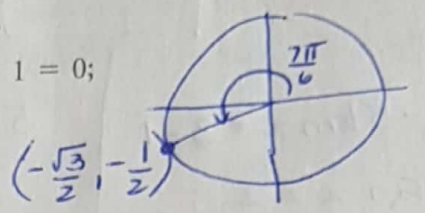
$(\sqrt{3})^2 - 3$

$3 - 3$

$0 = 0$

LS = RS

b) $8\sin^2 x + 6\sin x + 1 = 0$; $x = \frac{7\pi}{6}$



LS | RS

$8(\sin^2 \frac{7\pi}{6}) + 6\sin(\frac{7\pi}{6}) + 1$ | 0

$8(-\frac{1}{2})^2 + 6(-\frac{1}{2}) + 1$

$8(\frac{1}{4}) - 3 + 1$

$2 - 3 + 1$

0 | 0

LS = RS

B

6. Solve each equation over the domain $0 \le x < 2\pi$, then state the general solution.

a) $3\cos x - 2 = 0$

$3\cos x - 2 = 0$

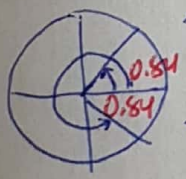
$3\cos x = 2$

$\cos x = \frac{2}{3}$

$x = \cos^{-1}(\frac{2}{3})$

$x = x_R = 0.84^r$

Note: $\cos x$ is +ve in QI & QIV.



x in QI: $x = 0.84$

x in QIV: $x = 2\pi - 0.84$

$= 5.44^r$

General solution.

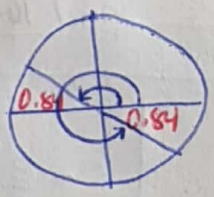
$0.84 + 2\pi k$, $k \in \mathbb{Z}$

and $5.44 + 2\pi k$, $k \in \mathbb{Z}$

b) $2\tan x + \sqrt{5} = 0$

$2\tan x = -\sqrt{5}$

$\tan x = -\frac{\sqrt{5}}{2}$



where is $\tan x$ -ve?

In QII and QIV

the reference angle:

$\tan x = \frac{\sqrt{5}}{2}$

$x = \tan^{-1}(\frac{\sqrt{5}}{2})$

$x = 0.84^r$

in QII: $x = \pi - 0.84$

$= 2.30^r$

in QIV: $x = 2\pi - 0.84$

$= 5.44^r$

General solution

$2.30 + 2\pi k$, $k \in \mathbb{Z}$

and $5.44 + 2\pi k$, $k \in \mathbb{Z}$.

← DOMAIN

7. Solve each equation for $-\pi \le x \le \pi$.

a) $3\tan x - 3 = 5\tan x - 1$

$3\tan x - 5\tan x = 3 - 1$

$-2\tan x = 2$

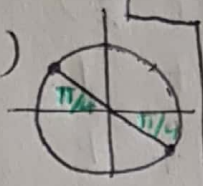
$\tan x = -1$

$\tan x$ is -ve in QII & QIV

$\tan x_R = +1$

$x_R = \tan^{-1}(+1)$

$x_R = \frac{\pi}{4}$



∴ in QII: $x = \frac{3\pi}{4}$

in QIV: $x = -\frac{\pi}{4}$

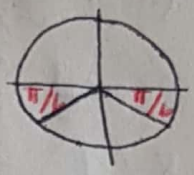
b) $5(1 + 2\sin x) = 2\sin x + 1$

$5 + 10\sin x = 2\sin x + 1$

$8\sin x = -4$

$\sin x = -\frac{1}{2}$ Where is $\sin x$ -ve? In QIII & QIV.

$\sin x_R = \frac{1}{2}$



$x_R = \sin^{-1}\left(\frac{1}{2}\right)$

$x_R = \frac{\pi}{6}$

In QIII: $x = -\frac{5\pi}{6}$

In QIV: $x = -\frac{\pi}{6}$

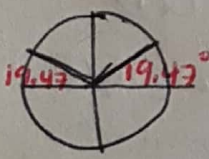
8. a) Solve each equation for $-180° \le x \le 90°$. D: $[-180, 0] \cup [0, 90]$

i) $2\csc x = 6$

$\csc x = \frac{6}{2}$

$\csc x = 3$

$\sin x = \frac{1}{3}$ sine is +ve in QI & QII



$x_R = \sin^{-1}\left(\frac{1}{3}\right)$

$x_R = 19.47°$

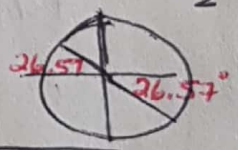
∴ $x = 19.47°$

ii) $-6 = 3\cot x$

$3\cot x = -6$

$\cot x = -2$

$\tan x = -\frac{1}{2}$ tan is -ve in QII & QIV



$x_R = \tan^{-1}\left(\frac{1}{2}\right)$

$x_R = 26.57°$

∴ $x = -26.57°$

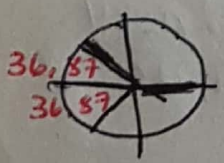
b) Solve each equation for $-90° \le x \le 180°$. D: $[-90, 0] \cup [0, 180]$

i) $4\sec x = -5$

$\sec x = -\frac{5}{4}$

$\cos x = -\frac{4}{5}$

$\cos x$ is -ve in QII & QIII



$x_R = \cos^{-1}\left(\frac{4}{5}\right)$

$x_R = 36.87°$

∴ In QII: $x = 180 - 36.87$

$x = 143.13°$

ii) $-\frac{1}{2} = \frac{1}{3}\csc x$

$-\frac{3}{2} = \csc x$

$-\frac{2}{3} = \sin x$

$\sin x$ is -ve in QIII & QIV

$x_R = \sin^{-1}\left(\frac{2}{3}\right)$

$x_R = 41.81°$

In QIV: $-41.81° = x$